The analysis of streamflow prediction uncertainty of CREW model using GLUE

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Predictive uncertainty =

Input uncertainty
+ Parameter uncertainty
+ Model structure uncertainty
+ Initial condition uncertainty
How?
- Construct prediction bands
- Monte Carlo Simulation

Why?
- To assess overall model prediction ability
- To learn the way to reduce prediction uncertainty
Outline

1. CREW: a physically based and distributed hydrological model at the catchment scale
2. Study area: Howard springs and Susannah brook
3. The analysis of model uncertainty in streamflow prediction
   3.1 construction of uncertainty bands in streamflow prediction
   3.2 the value of additional data: uncertainty quantification and reduction
1. CREW

A physically based and distributed hydrological model at the catchment scale
CREW  (Lee et al., 2006)

- Balance of mass and momentum at the scale of catchment: Reggiani (1998, 1999)

- Benefits: physically sound, less input data requirement, less computational cost, suitable for large scale modeling (~ > 100 km²)

- Application: Weiherbach, Germany  (Lee et al., 2006)  
  Collie river basin, Australia  (Lee et al., 2006)
Reggiani et al.’s theory

Spatial scale

Basin-scale theory and modeling

Micro-scale theory and modeling

Freeze & Harlan (1969)


- [http://www.fsl.orst.edu/lter/research/component/hydro/summary.cfm?sum=dye02&topnav=62](http://www.fsl.orst.edu/lter/research/component/hydro/summary.cfm?sum=dye02&topnav=62)
- Yoshi (2003)
Discretization: 1 catchment ➔ 13 analysis units

Reggiani et al. (1998)
Water Balance Model (CREW)

Concentrated overland flow zone

Saturated overland flow zone

Unsaturated zone

Saturated zone

Channel reach

e_{ij}^{ext}: exchange mass fluxes between i and j zone

### Governing equations (CREW)

- **Mass balance equations** (Reggiani et al., 1998, 1999; Lee et al., 2006)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Equation</th>
</tr>
</thead>
</table>
| Unsaturated Zone (U)      | \[
\varepsilon \frac{d}{dt} (y^s \omega^s s^u) = \min \left[ i \omega_n K_s + \alpha \frac{K_s |\Psi(1-s^u)\epsilon}{s^u y^s} \right] - \varepsilon \omega^s y^s \min \left[ e_p + M K_s \left( \frac{s^u}{(1-s^u)^{0.5}} \right) \left( \frac{s^u}{m} \right) \right] 
\] |
| Saturated Zone (S)        | \[
\varepsilon \frac{d}{dt} (y^s) = \varepsilon \omega^s y^s \frac{K_s |\Psi|}{s^u Z} + \frac{y^s}{Z} \frac{K_s |\Psi|}{s^u y^s} \] |
| Concentrated Overland Flow Zone (C) | \[
\frac{d}{dt} (y^c \omega^c) = \omega^c J - \min [i \omega_n, \frac{K_s |\Psi(1-s^u)\epsilon}{s^u y^c}] 
\] |
| Saturated Overland Flow Zone (O) | \[
\frac{d}{dt} (y^o \omega^o) = \omega^o K_s \left( \frac{y^o s^o \omega^o + y^o}{Z |\Psi|} \right) + \alpha \frac{y^o}{s^o \omega^o} + \omega^o J - \alpha^o K_s \left( \frac{y^o}{s^o \omega^o} \right) 
\] |
| Channel Reach Zone (R)    | \[
\frac{d}{dt} (m^r \xi^r) = \alpha^o \frac{y^o}{s^o \omega^o} + q_s + \sum \frac{m^i \xi^i}{\text{inflow}} - \sum \frac{m^i \xi^i}{\text{outflow}} + \xi^w J 
\] |
**Governing equations (CREW)**

- **Momentum balance equations** (Reggiani et al., 1998, 1999, 2000)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Equation</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsaturated Zone (U)</td>
<td>[ v_{z}^{u} = \bar{K} \left[ -s^{u} + \frac{1}{2} \frac{\left</td>
<td>\Psi \right</td>
</tr>
<tr>
<td>Saturated Zone (S)</td>
<td>ignoring</td>
<td></td>
</tr>
<tr>
<td>Concentrated Overland Flow Zone (C)</td>
<td>[ v^{c} = \frac{1}{n_{m}} \left[ y^{c} \right]^{3/2} \left[ \sin(y^{c}) \right]^{1/2} ]</td>
<td>The catchment scale Manning eq.</td>
</tr>
<tr>
<td>Saturated Overland Flow Zone (O)</td>
<td>[ v^{o} = \frac{1}{n_{m}} \left[ y^{o} \right]^{3/2} \left[ \sin(y^{o}) \right]^{1/2} ]</td>
<td>The catchment scale Manning eq.</td>
</tr>
<tr>
<td>Channel Reach Zone (R)</td>
<td>[ v^{r} = \frac{1}{n_{m}} \sqrt{ \left[ \frac{R^{r}}{P^{r}} \right]^{3/2} \left[ m^{r} \sin(y^{r}) \right] \pm \sum_{r} \left{ \frac{1}{4} y^{r} (m^{r} + m^{r}) \cos \delta_{l} \right} - \frac{1}{2} y^{r} m^{r} } ]</td>
<td>The catchment scale Diffusion wave eq.</td>
</tr>
</tbody>
</table>
2. Study area

Howard springs

Susannah brook

Photo: Carlos Ocampo

Stan Skymanski (2006)
Howard springs

- Area: 126 km²
- Annual rainfall: R = 2256 [mm/yr]
- Annual potential evapotranspiration: \( E_p = 2238 [\text{mm/yr}] \)
- Monthly rainfall
- Monthly potential evapotranspiration
- Mean monthly flux
- Mean annual flux

\[ RC = \frac{Q}{R} = 0.33 - 0.48 \text{ (wet season)} \]
\[ DI = \frac{E_p}{R} = 0.99 \]
Susannah brook

23 km²
R=872 [mm/yr]
Pan E=2130 [mm/yr]
RC=Q/R=0.17
DI=E/R=2.5
3. The analysis of model uncertainty in streamflow prediction

3.1. construction of uncertainty bands in streamflow prediction
**Generalized Likelihood Uncertainty Estimation (GLUE; Beven and Binley, 1992)**

- A Bayesian Monte-Carlo simulation-based technique
  - Likelihood measure:
    \[
    L(\theta_i|Y) = 1 - \frac{\sigma_i^2}{\sigma_o^2} \quad \left(\sigma_i^2 < \sigma_o^2\right)
    \]
    \[
    \begin{align*}
    \theta_i & : \text{parameter set} \quad Y : \text{data} \\
    \sigma_i^2 & : \text{error variance} \quad \sigma_o^2 : \text{observed variance}
    \end{align*}
    \]
  - Parameter sets: 30,000
Susannah brook: 99% Uncertainty bounds in runoff prediction

1997

1998

1999

2000
Howard springs: 99% Uncertainty bounds in runoff prediction

Data set 1 (1997/Sep – 1998/Sep)

Data set 2 (1998/Sep – 2000/Sep)

Data set 3 (2000/Sep – 2001/Sep)

Data set 4 (2001/Sep – 2003/Mar)
Prediction uncertainty in streamflow

Susannah brook

Observed streamflow

Howard springs

Observed streamflow
Prediction uncertainty in annual water balance

I : infiltration
R : recharge
IE : infiltration excess
SE : saturation excess
SS : subsurface flow
E : simulated evaporation
Q : simulated streamflow
3.2 the value of additional data
Flux data as additional data
Hutley et al., (2000, 2005)
The use of evaporation data (2001/1/1 – 2003/3/29)

At the beginning of the rainy season, low flows are sensitive to evaporation process.

\[ K(s) \]

Soil moisture content (s)

0

Dry season

Rainy season
Uncertainty (H): measure & reduction

\[ H = - \sum L_i \log_2 L_i \]

Shannon entropy measure (1948a, b)
4. Summary

- Regarding *uncertainty in streamflow prediction*

1. Uncertainty analysis using GLUE revealed *poor CREW performance* at peak flows.

2. The use of flux data helped *reduce* uncertainties in streamflow prediction which were *quantified* by Shannon entropy.
4. Summary

- Regarding what we learn from uncertainty analysis

1. Through the simulation of Susannah brook and Howard springs using CREW with GLUE showed that uncertainty bounds of streamflow were related to annual water balances of catchments.

2. At the simulation of Howard springs, low flows are sensitive to the changes in evaporation process at the beginning of the rainy season, but insensitive at the end of rainy season due to the nonlinear control of soil with respect to water movement.
Thank you!!!